PHYSICS NYB-10/11 Winter 2007

Lecture 12: Kirchhoff's rules

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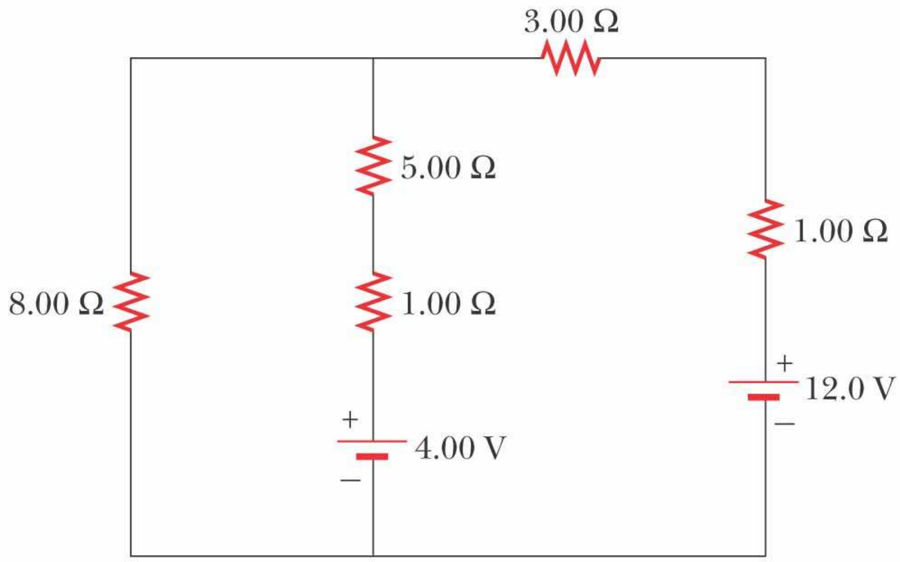
Review

- $I = \dfrac{dQ}{dt}$
- $I = \dfrac{\Delta V}{R}$
- $P = \Delta V I = RI^2 = \dfrac{\Delta V^2}{R}$
- Resistors in series: $R_{eq} = \sum_i R_i$,

  $I_1 = I_2 = I_3 = ...$
- Resistors in parallel: $\dfrac{1}{R_{eq}} = \sum_i \dfrac{1}{R_i}$,

  $\Delta V_1 = \Delta V_2 = \Delta V_3 = ...$

Review

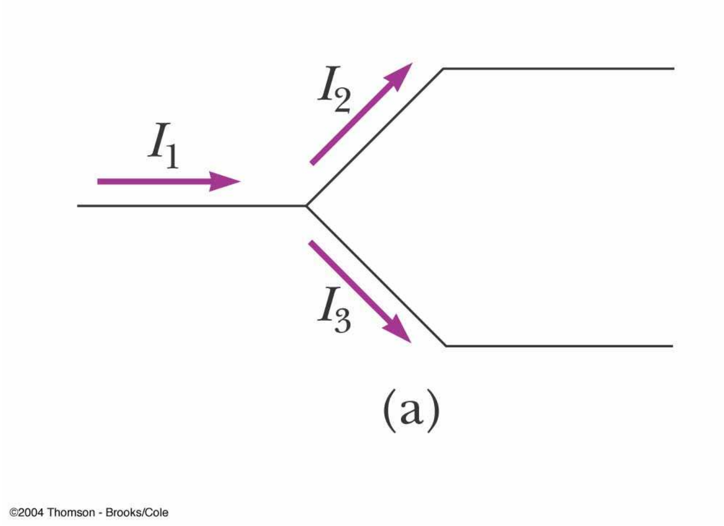


- How can we reduce the following circuit?
- First, notice that the two resistors on the middle wire are in series...
- Then the two resistors on the right wire are also in series...
- But then something is wrong... We can't reduce further.

Kirchhoff's rules

Often, resistors in circuits are not simply in parallel or series. *Kirchhoff's rules* will allow us to solve such circuits.

Junction rule: this rule simply states what we already know, namely that current has to be conserved. So the total current coming into a juntion has to equal the total current coming out.

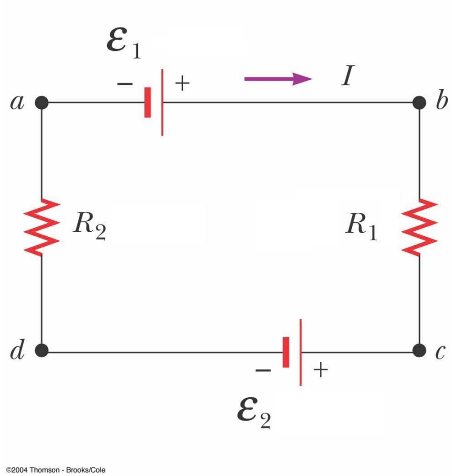


$$I_1 = I_2 + I_3$$

Kirchhoff's rules

Often, resistors in circuits are not simply in parallel or series. *Kirchhoff's rules* will allow us to solve such circuits.

Loop rule: When go completely around a loop in a circuit, you end up at the same point you started at, so every change in the potential you encountered along the loop better add up to zero!



$$\sum \Delta V = \mathcal{E}_1 - IR_1 - \mathcal{E}_2 - IR_2 = 0$$

Applying Kirchhoff's rules

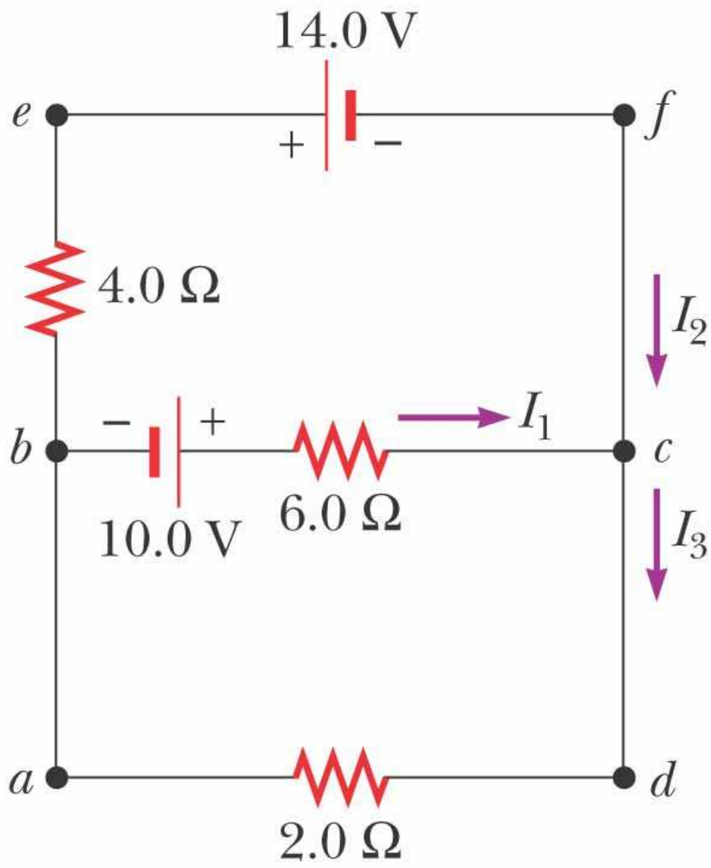
- Choose a direction for each current arbitrarily
- (If the direction you chose was wrong, you'll get a negative answer for that current)
- Write down conservation of current for each junction
- Write down $\sum \Delta V = 0$ for as many loops as are needed to include every component of the circuit at least once

Applying Kirchhoff's rules

- The potential drops in the direction of the current at a resistor
- The potential increases from negative end to positive end at a battery
- You will end up with *at least* as many equations as unknowns
- Grind away!

Examples

Find the currents in this circuit.



Examples

First, conservation of current at junction c tells us that

$$I_1 + I_2 = I_3$$

.

Then, we can make two loops, one in the upper part, one in the lower part. We'll follow them both in the clockwise direction. The loop rule then tells us in the bottom part that

$$+10 - 6I_1 - 2I_3 = 0$$

while in the top part we have

$$-4I_2 - 14 + 6I_1 - 10 = 0$$

This gives us our three equations, allowing us to solve for $I_1,\ I_2$ and I_3.

Examples

$$I_1 + I_2 = I_3$$

$$+10 - 6I_1 - 2I_3 = 0$$

$$-4I_2 - 14 + 6I_1 - 10 = -4I_2 + 6I_1 - 24 = 0$$

$$10 - 6I_1 - 2(I_1 + I_2) = 10 - 8I_1 - 2I_2 = 0$$

$$10 - 8I_1 - 2(-6 + 3/2I_1) = 22 - 11I_1 = 0$$

$$\boxed{\Rightarrow I_1 = 2 \text{ A}}$$

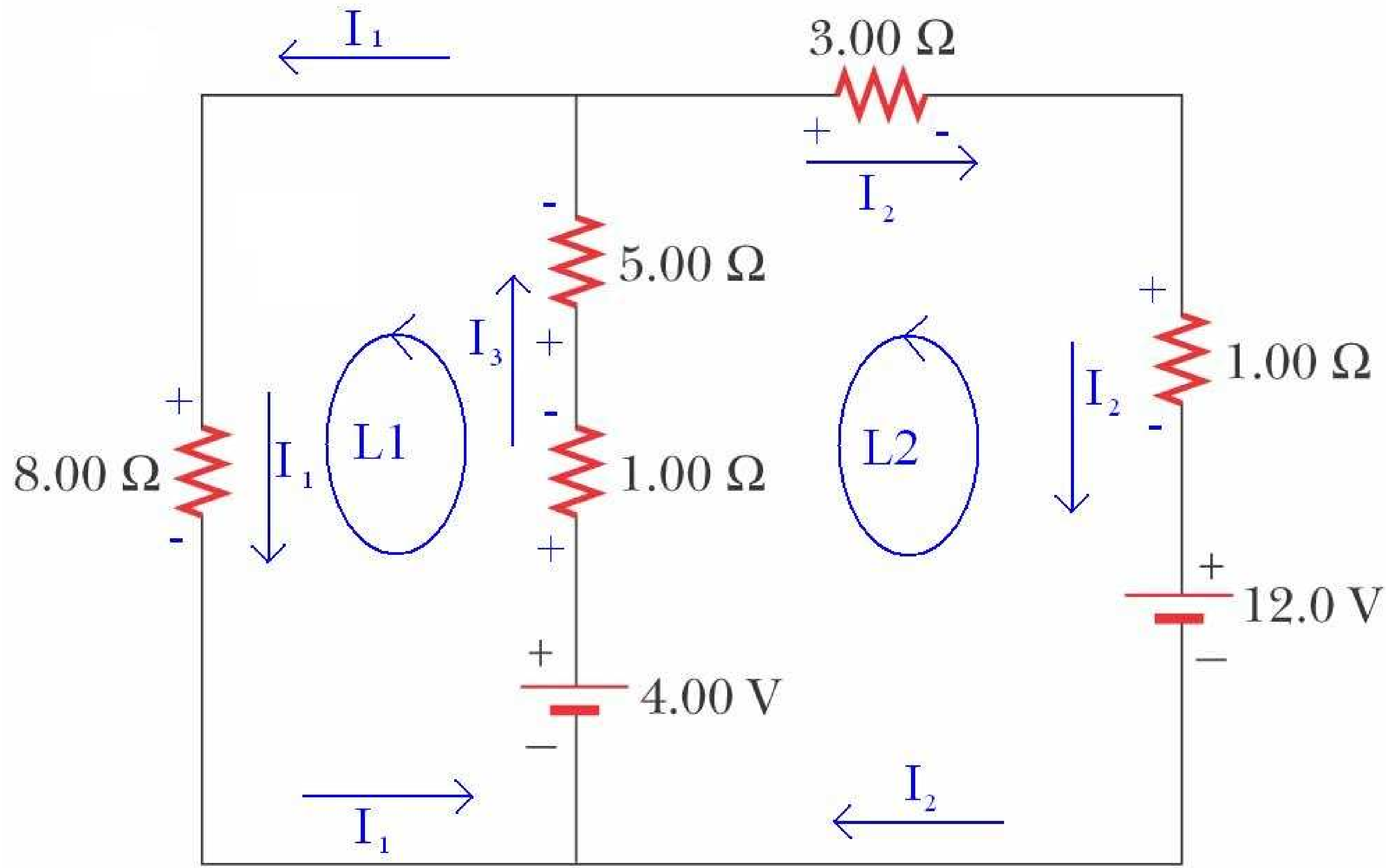
$$10 - 12 - 2I_3 = 0$$

$$\boxed{\Rightarrow I_3 = -1 \text{ A}}$$

$$\boxed{I_2 = -3 \text{ A}}$$

Examples

Find the currents in this circuit.



Examples

Applying the junction rule leads to

$$I_1 + I_2 = I_3$$

while the first loop gives us

$$4 - I_3 - 5I_3 - 8I_1 = 4 - 6I_3 - 8I_1 = 0$$

and the second loop tells us

$$12 + I_2 + 3I_2 + I_3 + 5I_3 - 4 = 8 + 4I_2 + 6I_3 = 0$$

We can sub for I_3 in the first loop by using the junction rule to find

$$4 - 6(I_1 + I_2) - 8I_1 = 4 - 14I_1 - 6I_2 = 0$$

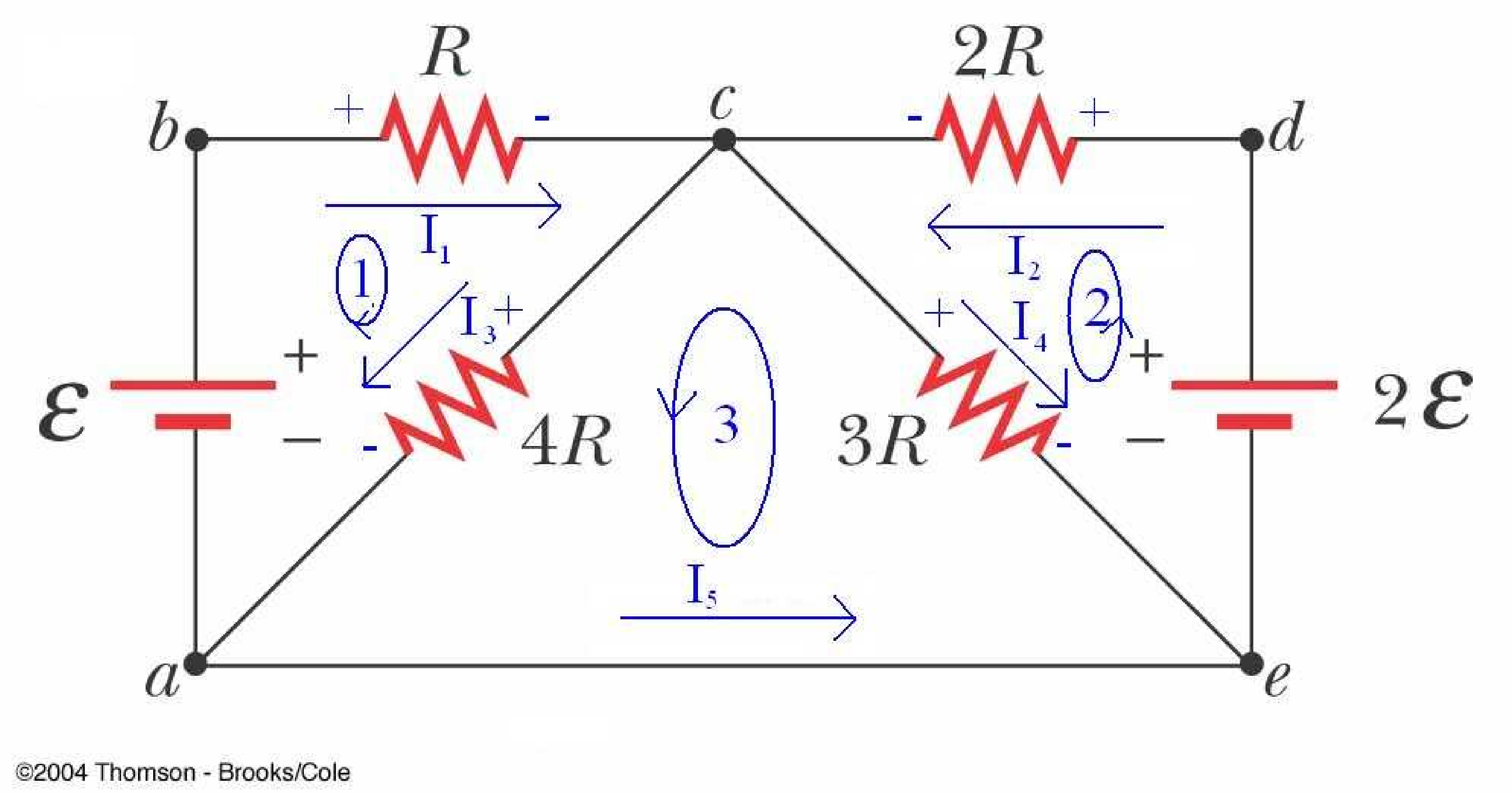
We can also do this in the second loop to obtain

$$8 + 6I_1 + 10I_2 = 0$$

We can now combine these two equations to get rid of I_2 and find $I_1 = 44/52 = 846$ mA. We plug this back into the equations and get $I_2 = -1.31$ A, and finally $I_3 = I_1 + I_2 = -464$ mA.

Examples

Find the currents in this circuit, where $\mathcal{E} = 250V$ and $R = 1$ kΩ.



Examples

For loop 1, we can write

$$\mathcal{E}/R - I_1 - 4I_3 = 0$$

For loop 2,

$$2\mathcal{E}/R - 2I_2 - 3I_4 = 0$$

For loop 3,

$$-4I_3 + 3I_4 = 0 \Rightarrow I_3 = \frac{3}{4}I_4$$

And the junction rule leads to

$$I_1 + I_2 = I_3 + I_4$$

and

$$I_3 = I_1 + I_5$$

.

Examples

Using the third of these, we can replace I_3 in all the others to get

$$\mathcal{E}/R - I_1 - 3I_4 = 0$$
$$2\mathcal{E}/R - 2I_2 - 3I_4 = 0$$
$$I_1 + I_2 = \frac{7}{4}I_4$$
$$\frac{3}{4}I_4 = I_1 + I_5$$

We can get rid of I_4 using the third of these four equations, leading to

$$\mathcal{E}/R - I_1 - 3\frac{4}{7}(I_1 + I_2) = \mathcal{E}/R - \frac{19}{7}I_1 - \frac{12}{7}I_2 = 0$$
$$2\mathcal{E}/R - 2I_2 - 3\frac{4}{7}(I_1 + I_2) = 2\mathcal{E}/R - \frac{26}{7}I_2 - \frac{12}{7}I_1 = 0$$
$$7I_5 = 3I_2 - 4I_1$$

Examples

We can combine the first two of these to get rid of $\mathcal{E}/R$ and find that

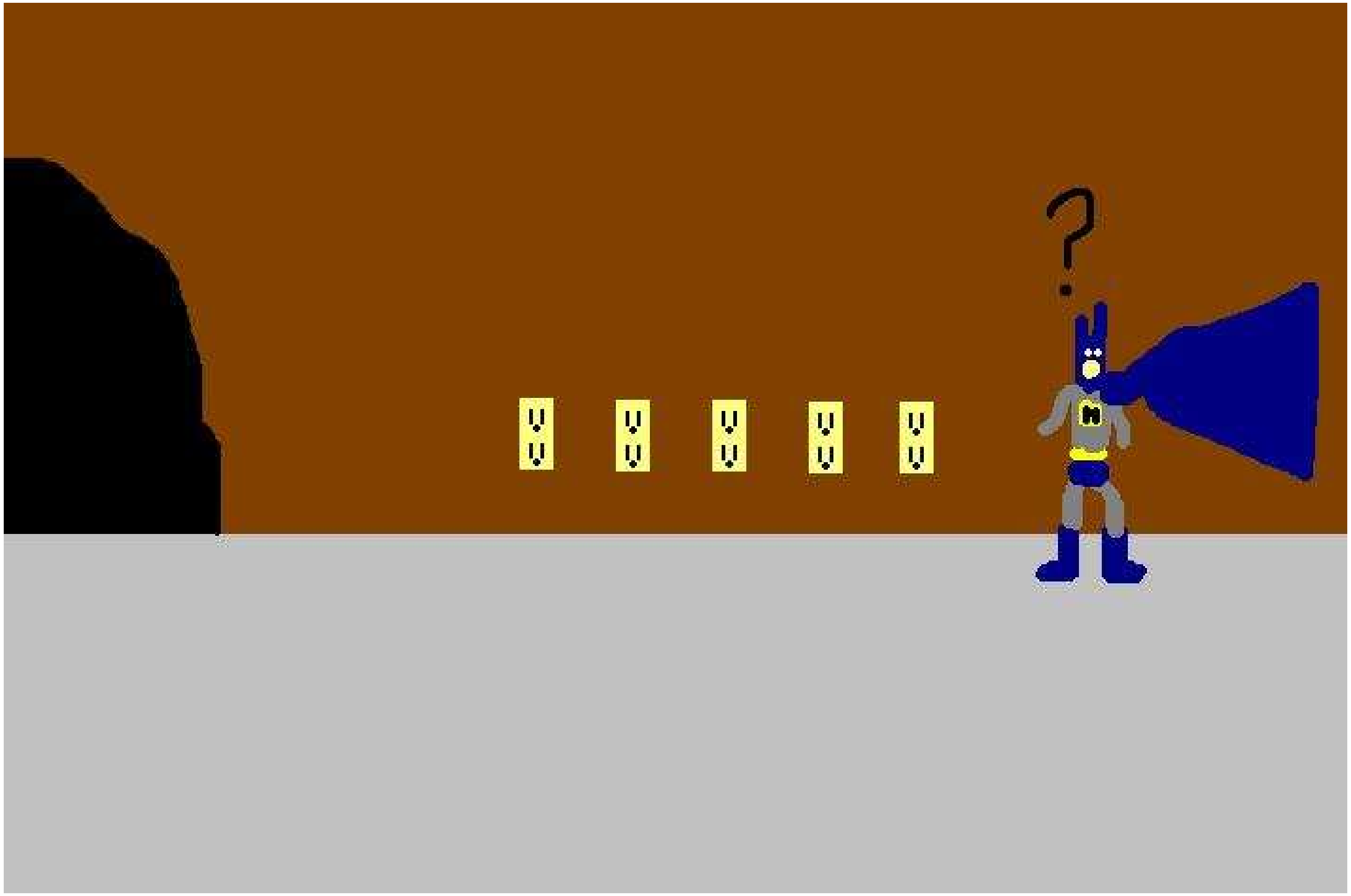
$$I_2 = 13I_1$$

and plugging this back into the equations leads to $I_1 = \frac{\mathcal{E}}{25R}$, $I_5 = 5I_1$, $I_4 = 8I_1$ and $I_3 = 6I_1$. Plugging numbers in, we get in the end,

$$I_1 = 10 \text{ mA}$$
$$I_2 = 130 \text{ mA}$$
$$I_3 = 60 \text{ mA}$$
$$I_4 = 80 \text{ mA}$$
$$I_5 = 50 \text{ mA}$$

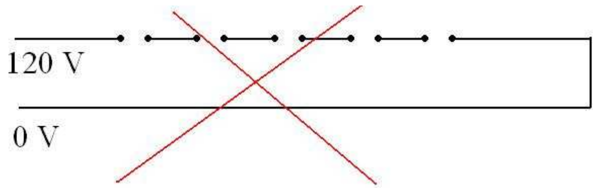
Household wiring

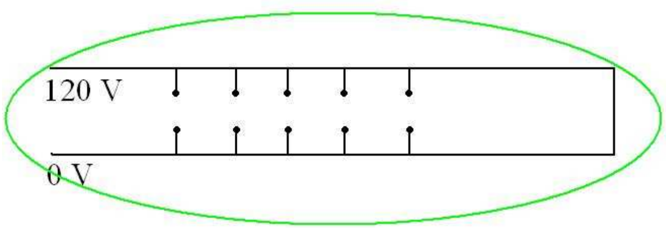
Batman needs to redo the wiring in the Batcave. He wants to have 5 different power sockets, to plug in his BatAppliances. Should he wire them in series or parallel? Should he include a circuit breaker?



Household wiring

Clearly, the circuit should be in parallel. This is for two reasons. Appliances in North-America are made to work under a 120 V potential difference, which is what the wires from the power company provide. If the circuit is in series, the potential difference across each socket will be different. Also, the circuit will only work when an appliance is plugged into each socket and is turned on. In contrast, if the sockets are in parallel, they all have 120 V across them, and current can pass through any single one even if the others are not in use.





Household wiring

The purpose of a circuit breaker is to create a break in the circuit when the current becomes too large. Indeed, if too much current is passing through the circuit's wires, the wires heat up (remember, wires *do* have a small non-zero resistance). The breaker's job is to switch off all current when the current exceeds some pre-determined value so the wires never overheat.

Household wiring

Batman is annoyed by the third prong on his appliances and wants to cut it off. Robin claims that might not be such a good idea. Who's correct?

Household wiring

The purpose of this wire is to *ground* the appliance. When the appliance is working properly, this wire is useless. However, if the wires in the appliance get used, they can sometimes come into contact with the casing. When this happens, the person holding the appliance is now part of the circuit... There is now a path for current to go from the 120 V wire to the 0 V ground *through the person holding the appliance*. That's a very bad thing! Indeed, it only takes a current of 100 mA to kill someone! To prevent this happening, a wire is connected between the casing and the ground. This way, if the 120 V wire comes into contact with the casing, almost all the current will pass through the low resistance grounded wire rather than anyone holding the appliance. So Batman should definitely *not* fiddle around with the third prong!

Assignment 5

- Chapter 28, problems 2, 14, 20, 30, 56, 63, 67

What to read for next lecture

- 23.5, 25.5